



Cubic Transformation Kappa Distribution: Properties and Applications

Israa Faleh Fadhil Omran

Department of Business Administration, College of Administration and Economics, University of Babylon, Babylon, Iraq

* Corresponding Author: **Israa Faleh Fadhil Omran**

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Abstract

In this paper, a Cubic Transmuted Kappa distribution (CTKD) to expand the work of cubic transmuted distribution families CTKD increases the ability of the transmuted distributions to be flexible and facilitate the modelling of more complex data. Some statistical properties of CTKD are discussed. To estimate the parameters for model, the maximum likelihood estimation technique in order to appear the interest of the Cubic Transmuted Kappa Distribution, two real datasets are used to examine the ability to apply it and to observe the performance of estimation techniques on CTKD, Kappa and transmuted Kappa distributions. The observed results showed that CTKD gives better fit than Kappa and transmuted Kappa distributions for the applied data sets.

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1. Introduction

The Statistical Researcher faces many statistical difficulties during the development of analyzing the data as well as Estimating the parameters related to the distribution, and among those problems that the researcher directs is the process of determining the appropriate and appropriate distribution of the data singularity. The Researchers Specialized in the ground specialized in the Statistical field have functioned to progress, the Probability Distributions and move them to the phase of the Transformed probability Distributions, in order to obtain, the best representation of the data with the least errors, Particularly, when the Researcher appearances the Problem of Choosing The Sample With an Equal Probability, which Made The Original Distribution Limited to the Modelling of Phenomena It is unserviceable and when it develops necessary, to suggest a Specific Modification to be Done finished the Transformed System in Order to Obtain, Vocabulary That Has The Same Appearance of Accidents, " now a Days Transmuted Distributions And Their Mathematical Properties are Widely Studied For Applied Sciences Experimental Data Sets. transmuted rayleigh distribution (merovci, 2013) ^[13], Transmuted Inverse, Rayleigh Distribution (Ahmad *et al.*, (2014) ^[2], Transmuted Generalized Inverse weibull distribution (khan and king, 2013) ^[9], transmuted modified inverse weibull distribution (elbatal, 2013) ^[8], transmuted log-logistic distribution (aryal, (2013) ^[4], transmuted modified weibull dsistribution and transmuted lomax distribution (ashour Aand eltehiwy, (2013) ^[5], transmuted frechet "distribution (mahmoud & mandouh, (2013) ^[12], transmuted pareto distribution (merovci & puka, (2014), transmuted generalized gamma distribution (lucena *et al.*, (2015) ^[11], transmuted weibull lomax distribution (afify *et al.*, (2015) ^[1] are reported with their various structural properties including explicit expressions for the Moments, Quantiles, Entropies, Mean Deviations And Order Statistics. all the Above Transmuted Distributions are Derived By Using quadratic Rank transmutation map(qRTM) Studied By Shaw & Buckley (2007) ^[16]. Report reveals that some properties of these distributions along with their parameters Are Estimated By Using Maximum Likelihood And bayesian Methods. usefulness Of some of These New Distributions Are Also illustrated With Experimental Data Sets. transmuted gumbel distribution (tGD) Along With Several Mathematical Properties Has Studied By aryal and tsokos (2009) ^[3] Using quadratic rank transmutation map(qRTM)and Reported That(TGD) Can be Used to Model Climate Data. Therefore an Attempt Has Been Made to Developed transmuted exponentiated gumbel distribution (tEGD) Using exponentiated Gumbel distribution (EGD),and The quadratic rank transmutation map(qRTM). the parameters of The (TEGD) are Estimated by The Method Of Maximum Likelihood and Applied to The Water Quality Parameter Data Sets For Study The Usefulness Of The Model.

A Random Variable X is Said to Have a Cubic Transmuted Distribution if its Cumulative Distribution Function (CDF) is Given By:

$$F(x, \gamma_1, \gamma_2) = (1 - \gamma_1)G(x) + (\gamma_2 - \gamma_1)[G(x)]^2 - \gamma_2[G(x)]^3; \quad -1 < \gamma_1, \gamma_2 < 1 \quad (1)$$

Where $G(x)$ is the CDF of The Cubic Transmuted Distribution and $F(x)$ is the CDF OF The base Distribution. differentiating (1) it Gives The Probability Density Function (PDF) of The Transmuted Distribution as Zhao& Lu,. (2008) ^[17]:

$$f(x, \gamma_1, \gamma_2) = g(x)[\gamma_1 + 2(\gamma_2 - \gamma_1)G(x) + 3(1 - \gamma_2)[G(x)]^2] \quad (2)$$

Let X be the random variable having Kappa distribution with parameters (α, θ, β) , then the probability density function and cumulative distribution function of the distribution is given by":

$$f(x, \alpha, \theta, \beta) = \begin{cases} \frac{\alpha\theta}{\beta} \left(\frac{x}{\beta}\right)^{\theta-1} \left[\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}, & (\text{if } x > 0) \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$F(x, \alpha, \theta, \beta) = \begin{cases} \left[\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}}\right]^{\left(\frac{1}{\alpha}\right)}, & (\text{if } x > 0) \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

2. Cubic Transmuted Kappa Distribution

"The cubic transmuted Kappa distribution is useful to analyze complex data arising in the distribution of wealth. The cubic transmuted Kappa distribution is defined as follows; The CDF of a cubic transmuted Kappa distribution is obtained by using " (4) in (1) Zhao& Lu,. (2008) ^[17].

$$F(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = \int_0^x f(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) dx \quad (5)$$

$$F(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = \gamma_1 \left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}}\right)^{\left(\frac{1}{\alpha}\right)} + (\gamma_2 - \gamma_1) \left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}}\right)^{\left(\frac{2}{\alpha}\right)} + (1 - \gamma_2) \left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}}\right)^{\left(\frac{3}{\alpha}\right)} \quad (6)$$

The reasonable shapes of CDF cubic transmuted Kappa distribution

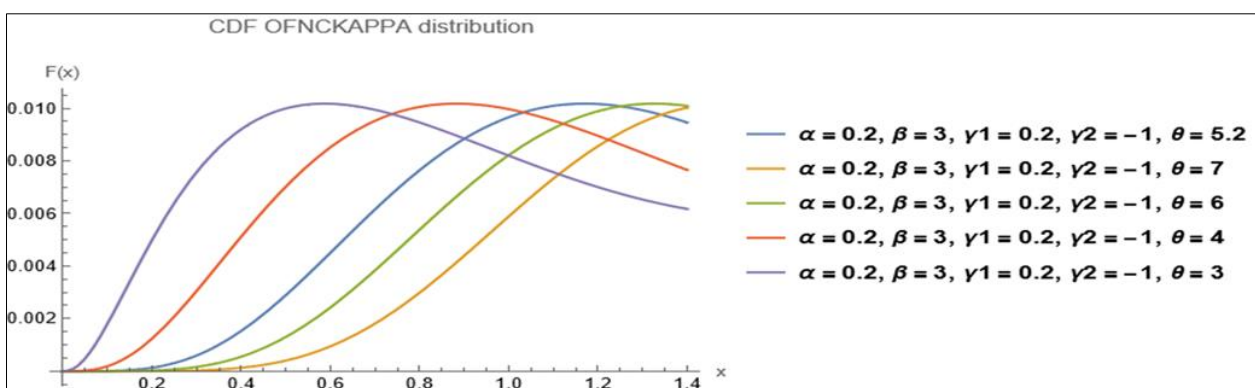


Fig 1: The Shape for the CDF cubic transmuted Kappa distribution

Suppose a continuous random variable X follows a Kappa distribution with parameters, then the density function of the cubic transmuted Kappa distribution is given by:-

$$f(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = \left(\frac{\alpha\theta}{\beta} [\gamma_1 \left(\frac{x}{\beta}\right)^{\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{-\left(\frac{\alpha+1}{\alpha}\right)} + 2(\gamma_2 - \gamma_1) \left(\frac{x}{\beta}\right)^{2\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{-\frac{\alpha-2}{\alpha}} + 3(1 - \gamma_2) \left(\frac{x}{\beta}\right)^{3\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{-\frac{\alpha-3}{\alpha}}] \right) \quad (7)$$

The reasonable shapes of PDF cubic transmuted Kappa distribution

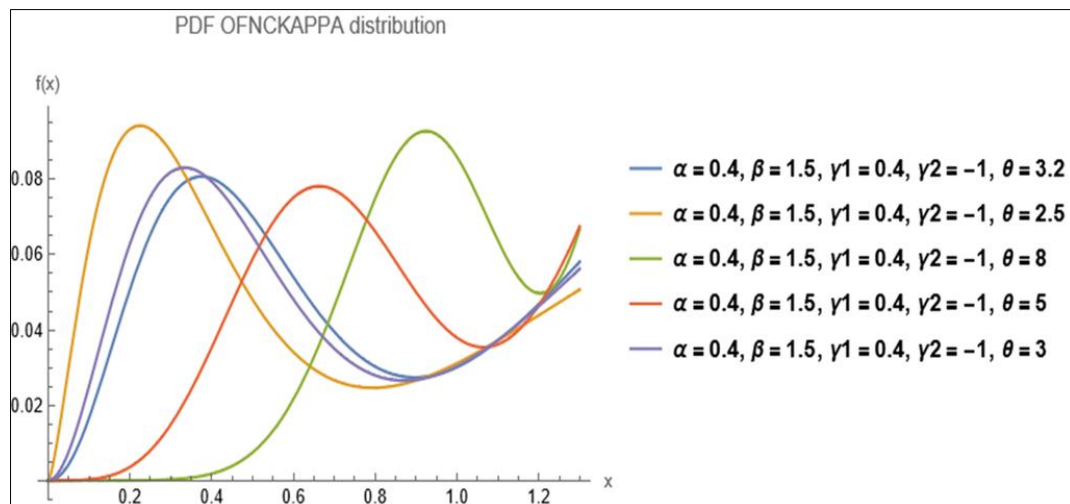


Fig 2: The Shape for the PDF cubic transmuted Kappa distribution

proof the room

$$\int_0^{\infty} f(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) \cdot dx = \int_0^{\infty} \left(\frac{\alpha\theta}{\beta} \gamma_1 \left(\frac{x}{\beta}\right)^{\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{-\frac{(\alpha+1)}{\alpha}} + 2(\gamma_2 - \gamma_1) \left(\frac{x}{\beta}\right)^{2\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{-\frac{(\alpha+2)}{\alpha}} + 3(1-\gamma_2) \left(\frac{x}{\beta}\right)^{3\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{-\frac{(\alpha+3)}{\alpha}} \right) \cdot dx$$

$$\text{let } c1 = \frac{\alpha\theta}{\beta} \gamma_1 \left(\frac{x}{\beta}\right)^{\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{-\frac{(\alpha+1)}{\alpha}}$$

$$\int_0^{\infty} c1 \cdot dx = \frac{\alpha\theta}{\beta} \gamma_1 \int_0^{\infty} \left(\frac{x}{\beta}\right)^{\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{-\frac{(\alpha+1)}{\alpha}} \cdot dx$$

$$= \gamma_1 \int_0^{\infty} \frac{\alpha\theta}{\beta} \left(\frac{x}{\beta}\right)^{\theta-1} \left[\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\frac{(\alpha+1)}{\alpha}} \cdot dx$$

$$\text{Let } u = \frac{x}{\beta}, x = u\beta, dx = \beta du$$

$$= \gamma_1 \int_0^{\infty} \frac{\alpha\theta}{\beta} (u)^{\theta-1} [\alpha + (u)^{\theta\alpha}]^{-\frac{(\alpha+1)}{\alpha}} \beta du$$

$$= \alpha\theta\gamma_1 \int_0^{\infty} (u)^{\theta-1} [\alpha + (u)^{\theta\alpha}]^{-\frac{(\alpha+1)}{\alpha}} du$$

$$\text{Let } z = (u)^{\theta\alpha} u = (z)^{\frac{1}{\theta\alpha}}, du = \frac{1}{\theta\alpha} (z)^{\frac{1}{\theta\alpha}-1} dz$$

$$= \alpha\theta\gamma_1 \int_0^{\infty} z^{-\frac{1}{\theta\alpha}} z^{\frac{1}{\alpha}} [\alpha + z]^{-\frac{(\alpha+1)}{\alpha}} \frac{1}{\theta\alpha} (z)^{\frac{1}{\theta\alpha}-1} dz$$

$$= \gamma_1 \alpha^{-\frac{(\alpha+1)}{\alpha}} \int_0^{\infty} z^{-1+\frac{1}{\alpha}} \left[1 + \frac{z}{\alpha}\right]^{-\frac{(\alpha+1)}{\alpha}} dz$$

$$\text{Let } y = \frac{z}{\alpha}, z = y\alpha, dz = \alpha dy$$

$$= \gamma_1 \frac{1}{\alpha} \int_0^{\infty} \frac{(y)^{-1+\frac{1}{\alpha}}}{[1+y]^{\frac{(\alpha+1)}{\alpha}}} dy$$

$$= \gamma_1 \frac{1}{\alpha} \frac{(\frac{1}{\alpha}-1)! (0)!}{(\frac{1}{\alpha})(\frac{1}{\alpha}-1)!}$$

$$\int_0^{\infty} c1. dx = \gamma_1$$

$$let c2 = \frac{\alpha\theta}{\beta} 2(\gamma_2 - \gamma_1) \left(\frac{x}{\beta}\right)^{2\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{\frac{-\alpha-2}{\alpha}}$$

$$\int_0^{\infty} c2. dx = \frac{\alpha\theta}{\beta} 2(\gamma_2 - \gamma_1) \int_0^{\infty} \left(\frac{x}{\beta}\right)^{2\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{\frac{-\alpha-2}{\alpha}}. dx$$

$$Let u = \frac{x}{\beta}, x = u\beta, dx = \beta du$$

$$= \alpha\theta 2(\gamma_2 - \gamma_1) \int_0^{\infty} (u)^{2\theta-1} (\alpha + (u)^{\theta\alpha})^{\frac{-\alpha-2}{\alpha}}. du$$

$$Let z = (u)^{\theta\alpha} u = (z)^{\frac{1}{\theta\alpha}}, du = \frac{1}{\theta\alpha} (z)^{\frac{1}{\theta\alpha}-1} dz$$

$$= \alpha\theta 2(\gamma_2 - \gamma_1) \int_0^{\infty} (u)^{2\theta-1} (\alpha + z)^{\frac{-\alpha-2}{\alpha}} \cdot \frac{1}{\theta\alpha} (z)^{\frac{1}{\theta\alpha}-1} dz$$

$$= \int_0^{\infty} c2. dx = (\gamma_2 - \gamma_1)$$

$$let c3 = \frac{\alpha\theta}{\beta} 3(1 - \gamma_2) \left(\frac{x}{\beta}\right)^{3\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{\frac{-\alpha-3}{\alpha}}$$

$$\int_0^{\infty} c3 dx = \frac{\alpha\theta}{\beta} 3(1 - \gamma_2) \int_0^{\infty} \left(\frac{x}{\beta}\right)^{3\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{\frac{-\alpha-3}{\alpha}} dx$$

$$Let u = \frac{x}{\beta}, x = u\beta, dx = \beta du$$

$$Let z = (u)^{\theta\alpha} u = (z)^{\frac{1}{\theta\alpha}}, du = \frac{1}{\theta\alpha} (z)^{\frac{1}{\theta\alpha}-1} dz$$

$$= 3(1 - \gamma_2) \alpha\theta \int_0^{\infty} ((z)^{\frac{1}{\theta\alpha}})^{3\theta-1} (\alpha + z)^{\frac{-\alpha-3}{\alpha}} \frac{1}{\theta\alpha} (z)^{\frac{1}{\theta\alpha}-1} dz$$

$$= 3(1 - \gamma_2) \int_0^{\infty} (z)^{\frac{3\theta-1}{\theta\alpha}} (\alpha + z)^{\frac{-\alpha-3}{\alpha}} (z)^{\frac{1-\alpha\theta}{\theta\alpha}} dz$$

$$= 3(1 - \gamma_2) \int_0^{\infty} (z)^{\frac{3\theta-1+1-\alpha\theta}{\theta\alpha}} (\alpha + z)^{\frac{-\alpha-3}{\alpha}} dz$$

$$\int_0^{\infty} c3 dx = (1 - \gamma_2)$$

$$\int_0^{\infty} f(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) dx = \int_0^{\infty} c1 + c2 + c3. dx$$

$$= \gamma_1 + (\gamma_2 - \gamma_1) + (1 - \gamma_2)$$

$$= 1$$

The Survival function of the cubic transmuted Kappa distribution given is:

$$S(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = 1 - F(x, \alpha, \theta, \beta, \gamma_1, \gamma_2)$$

$$S(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = 1 - \gamma_1 \left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}} \right)^{\left(\frac{1}{\alpha}\right)} + (\gamma_2 - \gamma_1) \left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}} \right)^{\left(\frac{2}{\alpha}\right)} + (1 - \gamma_2) \left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha + \left(\frac{x}{\beta}\right)^{\theta\alpha}} \right)^{\left(\frac{3}{\alpha}\right)} \quad (8)$$

The reasonable shapes Survival cubic transmuted Kappa distribution given is:

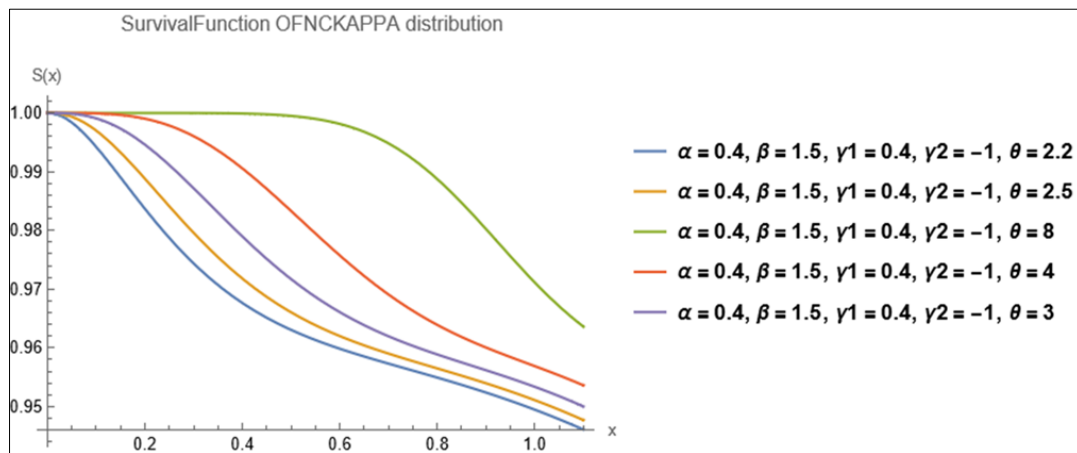


Fig 3: The Shape for the Survival cubic transmuted Kappa distribution

The hazard function of the cubic transmuted Kappa distribution given is:

For any random variable X which follows cubic transmuted Kappa distribution, its hazard function is given as:

$$h(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = \frac{f(x, \alpha, \theta, \beta, \gamma_1, \gamma_2)}{s(x, \alpha, \theta, \beta, \gamma_1, \gamma_2)}$$

$$h(\theta, \alpha) = \frac{\frac{V}{2} f\left(V \tan\left(\frac{\theta}{2}\right)\right) = \frac{V}{2} \frac{\alpha^2}{\alpha^2 + 1} \left(\alpha + V \tan\left(\frac{\theta}{2}\right)\right) e^{-\alpha \left(V \tan\left(\frac{\theta}{2}\right)\right) \sec^2\left(\frac{\theta}{2}\right)}}{\left(1 + \frac{\alpha V \tan\left(\frac{\theta}{2}\right)}{\alpha^2 + 1}\right) e^{-\alpha V \tan\left(\frac{\theta}{2}\right)}} \quad (9)$$

$$h(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = \frac{\frac{\alpha \theta}{\beta} \left[\gamma_1 \left(\frac{x}{\beta}\right)^{\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta \alpha}\right)^{-\frac{(\alpha+1)}{\alpha}} + 2(\gamma_2 - \gamma_1) \left(\frac{x}{\beta}\right)^{2\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta \alpha}\right)^{-\frac{\alpha-2}{\alpha}} + 3(1-\gamma_2) \left(\frac{x}{\beta}\right)^{3\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta \alpha}\right)^{-\frac{\alpha-3}{\alpha}} \right]}{1 - \gamma_1 \left(\frac{x}{\beta}\right)^{\frac{1}{\alpha}} + (\gamma_2 - \gamma_1) \left(\frac{x}{\beta}\right)^{\frac{2}{\alpha}} + (1-\gamma_2) \left(\frac{x}{\beta}\right)^{\frac{3}{\alpha}}} \quad (10)$$

3. Statistical Properties

In this section, some of the properties of the cubic transmuted Kappa distribution are discussed:

3.1. Moments

Let X mean the Random variable Follows cubic transmuted Kappa distribution then r^{th} order moment near origin of μ_r is: (afify et al., (2015) ^[1] "

$$E(X^r) = \int_0^\infty x^r f(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) \cdot dx \quad (11)$$

$$= \int_0^\infty x^r \frac{\alpha \theta}{\beta} \left[\gamma_1 \left(\frac{x}{\beta}\right)^{\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta \alpha}\right)^{-\frac{(\alpha+1)}{\alpha}} + 2(\gamma_2 - \gamma_1) \left(\frac{x}{\beta}\right)^{2\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta \alpha}\right)^{-\frac{\alpha-2}{\alpha}} + 3(1-\gamma_2) \left(\frac{x}{\beta}\right)^{3\theta-1} \left(\alpha + \left(\frac{x}{\beta}\right)^{\theta \alpha}\right)^{-\frac{\alpha-3}{\alpha}} \right] \cdot dx$$

$$E(X^r) = \left(\frac{\gamma_1 \beta^r \alpha^{\frac{r}{\alpha}} \Gamma\left[1 - \frac{r}{\alpha \theta}\right] \Gamma\left[\frac{r+\theta}{\alpha \theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} \right) + \left(\frac{2(\gamma_2 - \gamma_1) \beta^r \alpha^{-1 + \frac{r}{\alpha \theta}} \Gamma\left[1 - \frac{r}{\alpha \theta}\right] \Gamma\left[\frac{r+2\theta}{\alpha \theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} \right) + \left(\frac{3(1-\gamma_2) \beta^r \alpha^{-1 + \frac{r}{\alpha \theta}} \Gamma\left[1 - \frac{r}{\alpha \theta}\right] \Gamma\left[\frac{r+3\theta}{\alpha \theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right); r = 1, 2, 3 \dots n \quad (12)$$

Where $r=1$

$$E(X^1) = \left(\frac{\gamma_1 \beta^1 \alpha^{\frac{1}{\alpha}} \Gamma\left[1 - \frac{1}{\alpha \theta}\right] \Gamma\left[\frac{1+\theta}{\alpha \theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} \right) + \left(\frac{2(\gamma_2 - \gamma_1) \beta^1 \alpha^{-1 + \frac{1}{\alpha \theta}} \Gamma\left[1 - \frac{1}{\alpha \theta}\right] \Gamma\left[\frac{1+2\theta}{\alpha \theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} \right) + \left(\frac{3(1-\gamma_2) \beta^1 \alpha^{-1 + \frac{1}{\alpha \theta}} \Gamma\left[1 - \frac{1}{\alpha \theta}\right] \Gamma\left[\frac{1+3\theta}{\alpha \theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right)$$

Where $r=2$

$$E(X^2) = \left(\frac{\gamma_1 \beta^2 \alpha^{\frac{2}{\alpha}} \Gamma\left[1 - \frac{2}{\alpha \theta}\right] \Gamma\left[\frac{2+\theta}{\alpha \theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} \right) + \left(\frac{2(\gamma_2 - \gamma_1) \beta^2 \alpha^{-1 + \frac{2}{\alpha \theta}} \Gamma\left[1 - \frac{2}{\alpha \theta}\right] \Gamma\left[\frac{2+2\theta}{\alpha \theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} \right) + \left(\frac{3(1-\gamma_2) \beta^2 \alpha^{-1 + \frac{2}{\alpha \theta}} \Gamma\left[1 - \frac{2}{\alpha \theta}\right] \Gamma\left[\frac{2+3\theta}{\alpha \theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right)$$

Where $r=3$

$$E(x^3) = \left(\frac{\gamma_1 \beta^3 \alpha^{\frac{3}{\alpha\theta}} \Gamma\left[1 - \frac{3}{\alpha\theta}\right] \Gamma\left[\frac{3+\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} \right) + \left(\frac{2(\gamma_2 - \gamma_1) \beta^3 \alpha^{-1 + \frac{3}{\alpha\theta}} \Gamma\left[1 - \frac{3}{\alpha\theta}\right] \Gamma\left[\frac{3+2\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} \right) + \left(\frac{3(1-\gamma_2) \beta^3 \alpha^{-1 + \frac{3}{\alpha\theta}} \Gamma\left[1 - \frac{3}{\alpha\theta}\right] \Gamma\left[\frac{3+3\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right)$$

Where $r=4$

$$E(x^4) = \left(\frac{\gamma_1 \beta^4 \alpha^{\frac{4}{\alpha\theta}} \Gamma\left[1 - \frac{4}{\alpha\theta}\right] \Gamma\left[\frac{4+\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} \right) + \left(\frac{2(\gamma_2 - \gamma_1) \beta^4 \alpha^{-1 + \frac{4}{\alpha\theta}} \Gamma\left[1 - \frac{4}{\alpha\theta}\right] \Gamma\left[\frac{4+2\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} \right) + \left(\frac{3(1-\gamma_2) \beta^4 \alpha^{-1 + \frac{4}{\alpha\theta}} \Gamma\left[1 - \frac{4}{\alpha\theta}\right] \Gamma\left[\frac{4+3\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right)$$

3.2. Moments About the Mean

Let X mean The random variable Follows cubic transmuted Kappa distribution Then Moments About The Mean order Moment about origin of μ_r is:"

$$E(x - \mu)^r = \frac{\alpha\theta}{\beta} \int_0^\infty (x - \mu)^r f(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) \cdot dx \quad (13)$$

$$E(x - \mu)^r = \sum_{j=0}^r \binom{r}{j} (-\mu)^{r-j} x^j$$

$$E(x - \mu)^r = \left(\begin{array}{l} \frac{\sum_{j=0}^r \binom{r}{j} (-\mu)^{r-j} \gamma_1 \beta^j \alpha^{\frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} \\ + \sum_{j=0}^r \binom{r}{j} (-\mu)^{r-j} \frac{2(\gamma_2 - \gamma_1) \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+2\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} \\ + \frac{3(1-\gamma_2) \sum_{j=0}^r \binom{r}{j} (-\mu)^{r-j} \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+3\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \end{array} \right)$$

Where $r=2$

$$\begin{aligned} E(x - \mu)^2 &= \sum_{j=0}^2 \binom{2}{j} (-\mu)^{2-j} \left(\frac{\gamma_1 \beta^j \alpha^{\frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} + \frac{2(\gamma_2 - \gamma_1) \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+2\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} + \frac{3(-\mu)^{r-j} \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+3\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right) \\ \sigma^2 &= \sum_{j=0}^2 \binom{2}{j} (-\mu)^{2-j} \left(\frac{\gamma_1 \beta^j \alpha^{\frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} + \frac{2(\gamma_2 - \gamma_1) \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+2\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} + \frac{3(-\mu)^{r-j} \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+3\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right) \\ \sigma &= \sqrt{\sum_{j=0}^2 \binom{2}{j} (-\mu)^{2-j} \left(\frac{\gamma_1 \beta^j \alpha^{\frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} + \frac{2(\gamma_2 - \gamma_1) \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+2\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} + \frac{3(-\mu)^{r-j} \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+3\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right)} \end{aligned} \quad (14)$$

Where $r=3$

$$E(x - \mu)^3 = \sum_{j=0}^3 \binom{3}{j} (-\mu)^{r-j} \left(\frac{\gamma_1 \beta^j \alpha^{\frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} + \frac{2(\gamma_2 - \gamma_1) \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+2\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} + \frac{3(-\mu)^{r-j} \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+3\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right)$$

Where $r=4$

$$E(x - \mu)^4 = \sum_{j=0}^4 \binom{4}{j} (-\mu)^{4-j} \left(\frac{\gamma_1 \beta^j \alpha^{\frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} + \frac{2(\gamma_2 - \gamma_1) \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+2\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} + \frac{3(-\mu)^{r-j} \beta^j \alpha^{-1 + \frac{j}{\alpha\theta}} \Gamma\left[1 - \frac{j}{\alpha\theta}\right] \Gamma\left[\frac{j+3\theta}{\alpha\theta}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right)$$

3.3. Coefficients of Variation (Cv)

Coefficient of cubic transmuted Kappa distribution is given by(ashour Aand eltehiwy, (2013) [5],:

$$C.V = \frac{\sigma}{\mu} \times 100$$

$$C.V = \frac{\sqrt{\sum_{j=0}^2 \binom{2}{j} (-\mu)^{2-j} \left(\frac{\gamma_1 \beta^j \alpha^{\frac{j}{\alpha}} \Gamma\left[1-\frac{j}{\alpha}\right] \Gamma\left[\frac{j+\theta}{\alpha}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} + \frac{2(\gamma_2-\gamma_1) \beta^j \alpha^{-1+\frac{j}{\alpha}} \Gamma\left[1-\frac{j}{\alpha}\right] \Gamma\left[\frac{j+2\theta}{\alpha}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} + \frac{3(-\mu)^{r-j} \beta^j \alpha^{-1+\frac{j}{\alpha}} \Gamma\left[1-\frac{j}{\alpha}\right] \Gamma\left[\frac{j+3\theta}{\alpha}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right)}}{\left(\frac{\gamma_1 \beta^1 \alpha^{\frac{1}{\alpha}} \Gamma\left[1-\frac{1}{\alpha}\right] \Gamma\left[\frac{1+\theta}{\alpha}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} \right) + \left(\frac{2(\gamma_2-\gamma_1) \beta^1 \alpha^{-1+\frac{1}{\alpha}} \Gamma\left[1-\frac{1}{\alpha}\right] \Gamma\left[\frac{1+2\theta}{\alpha}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} \right) + \left(\frac{3(1-\gamma_2) \beta^1 \alpha^{-1+\frac{1}{\alpha}} \Gamma\left[1-\frac{1}{\alpha}\right] \Gamma\left[\frac{1+3\theta}{\alpha}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right)} \quad (15)$$

3.4. Coefficient of Skewedness (C.S)

COEFFICIENT OF SKEWEDNESS for cubic transmuted Kappa distribution is given by:

$$S.K = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}}$$

$$S.K = \frac{\sum_{j=0}^3 \binom{3}{j} (-\mu)^{3-j} \left(\frac{\gamma_1 \beta^j \alpha^{\frac{j}{\alpha}} \Gamma\left[1-\frac{j}{\alpha}\right] \Gamma\left[\frac{j+\theta}{\alpha}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} + \frac{2(\gamma_2-\gamma_1) \beta^j \alpha^{-1+\frac{j}{\alpha}} \Gamma\left[1-\frac{j}{\alpha}\right] \Gamma\left[\frac{j+2\theta}{\alpha}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} + \frac{3(-\mu)^{r-j} \beta^j \alpha^{-1+\frac{j}{\alpha}} \Gamma\left[1-\frac{j}{\alpha}\right] \Gamma\left[\frac{j+3\theta}{\alpha}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right)}{\left(\sum_{j=0}^2 \binom{2}{j} (-\mu)^{2-j} \left(\frac{\gamma_1 \beta^j \alpha^{\frac{j}{\alpha}} \Gamma\left[1-\frac{j}{\alpha}\right] \Gamma\left[\frac{j+\theta}{\alpha}\right]}{\Gamma\left[\frac{1}{\alpha}\right]} + \frac{2(\gamma_2-\gamma_1) \beta^j \alpha^{-1+\frac{j}{\alpha}} \Gamma\left[1-\frac{j}{\alpha}\right] \Gamma\left[\frac{j+2\theta}{\alpha}\right]}{\Gamma\left[\frac{2+\alpha}{\alpha}\right]} + \frac{3(-\mu)^{r-j} \beta^j \alpha^{-1+\frac{j}{\alpha}} \Gamma\left[1-\frac{j}{\alpha}\right] \Gamma\left[\frac{j+3\theta}{\alpha}\right]}{\Gamma\left[\frac{3+\alpha}{\alpha}\right]} \right) \right)^{\frac{3}{2}}} \quad (16)$$

3.5. Coefficient of Kurtosis (KS)

COEFFICIENT OF KURTOSIS (KS) for cubic transmuted Kappa distribution (ashour Aand eltehiwy, (2013) ^[6], is given by":

$$K.S = \frac{E(X - \mu)^4}{\varphi^4}$$

$$K.S = \frac{\sum_{j=0}^4 \binom{4}{j} (-\mu)^{4-j} \left(\frac{\alpha \beta + j \left(\Gamma\left[\frac{j+\beta}{\beta}\right] \right) \Gamma\left[1-\frac{j}{\beta}\right] \omega^j \varphi}{(\varphi+1) \alpha \beta} + \frac{\Gamma\left[1-\frac{j}{\beta}\right] \omega^j \varphi}{\varphi+1} \right)}{\left[\sum_{j=0}^2 \binom{2}{j} (-\mu)^{2-j} \left(\frac{\alpha \beta + j \left(\Gamma\left[\frac{j+\beta}{\beta}\right] \right) \Gamma\left[1-\frac{j}{\beta}\right] \omega^j \varphi}{(\varphi+1) \alpha \beta} + \frac{\Gamma\left[1-\frac{j}{\beta}\right] \omega^j \varphi}{\varphi+1} \right) \right]^2} \quad (17)$$

5. Parameter Estimation

The Method of Maximum Likelihood Estimate is Used for Estimating The Parameters of The Newly Proposed Distribution Known as of cubic transmuted Kappa distribution. Let $x_1, x_2, \dots, x_n$ be a Random Sample of Ssize n From of The cubic transmuted Kappa distribution, Then the Corresponding likelihood Function is Given By (Ahmad *et al.*, (2014) ^[2]: "

Let $x_1, x_2, x_3, x_4, \dots, x_n$ be a random sample of size n from cubic transmuted Kappa distribution The likelihood function, L cubic transmuted Kappa distribution. is given by:

$$L(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = f(x, \alpha, \theta, \beta, \gamma_1, \gamma_2)$$

$$L(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = \prod_{i=1}^n f(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) \quad (18)$$

$$L(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = \prod_{i=1}^n \left(\frac{\alpha \theta}{\beta} \left[\gamma_1 \left(\frac{x}{\beta} \right)^{\theta-1} \left(\alpha + \left(\frac{x}{\beta} \right)^{\theta \alpha} \right)^{-\frac{(\alpha+1)}{\alpha}} + 2(\gamma_2-\gamma_1) \left(\frac{x}{\beta} \right)^{2\theta-1} \left(\alpha + \left(\frac{x}{\beta} \right)^{\theta \alpha} \right)^{-\frac{\alpha-2}{\alpha}} \right. \right. \\ \left. \left. + 3(1-\gamma_2) \left(\frac{x}{\beta} \right)^{3\theta-1} \left(\alpha + \left(\frac{x}{\beta} \right)^{\theta \alpha} \right)^{-\frac{\alpha-3}{\alpha}} \right] \right)$$

$$\log L(x, \alpha, \theta, \beta, \gamma_1, \gamma_2) = \sum_{i=1}^n \ln \left(\frac{\alpha \theta}{\beta} \left[\gamma_1 \left(\frac{x}{\beta} \right)^{\theta-1} \left(\alpha + \left(\frac{x}{\beta} \right)^{\theta \alpha} \right)^{-\frac{(\alpha+1)}{\alpha}} + 2(\gamma_2-\gamma_1) \left(\frac{x}{\beta} \right)^{2\theta-1} \left(\alpha + \left(\frac{x}{\beta} \right)^{\theta \alpha} \right)^{-\frac{\alpha-2}{\alpha}} \right. \right. \\ \left. \left. + 3(1-\gamma_2) \left(\frac{x}{\beta} \right)^{3\theta-1} \left(\alpha + \left(\frac{x}{\beta} \right)^{\theta \alpha} \right)^{-\frac{\alpha-3}{\alpha}} \right] \right)$$

$$\frac{d \log L(x, \alpha, \theta, \beta, \gamma_1, \gamma_2)}{d\alpha} = \left\{ \sum_{i=1}^n \ln \left(\frac{\alpha \theta}{\beta} \left[\gamma_1 \left(\frac{x}{\beta} \right)^{\theta-1} \left(\alpha + \left(\frac{x}{\beta} \right)^{\theta \alpha} \right)^{-\frac{(\alpha+1)}{\alpha}} + 2(\gamma_2-\gamma_1) \left(\frac{x}{\beta} \right)^{2\theta-1} \left(\alpha + \left(\frac{x}{\beta} \right)^{\theta \alpha} \right)^{-\frac{\alpha-2}{\alpha}} \right. \right. \right. \\ \left. \left. + 3(1-\gamma_2) \left(\frac{x}{\beta} \right)^{3\theta-1} \left(\alpha + \left(\frac{x}{\beta} \right)^{\theta \alpha} \right)^{-\frac{\alpha-3}{\alpha}} \right] \right) \left[\frac{\partial f(x)}{\partial \alpha} \right] \right\} \quad (19)$$

$$\frac{d\log L(x, \alpha, \theta, \beta, \gamma_1, \gamma_2)}{d\theta} = \left\{ \sum_{i=1}^n \ln \left(\frac{\frac{\alpha\theta}{\beta} [\gamma_1 (\frac{x}{\beta})^{\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-(\frac{\alpha+1}{\alpha})} + 2(\gamma_2 - \gamma_1) (\frac{x}{\beta})^{2\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-\frac{\alpha-2}{\alpha}} + 3(1-\gamma_2) (\frac{x}{\beta})^{3\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-\frac{\alpha-3}{\alpha}}]} \right) \left[\frac{\partial f(x)}{\partial \theta} \right] \right\} \quad (20)$$

$$\frac{d\log L(x, \alpha, \theta, \beta, \gamma_1, \gamma_2)}{d\beta} = \left\{ \sum_{i=1}^n \ln \left(\frac{\frac{\alpha\theta}{\beta} [\gamma_1 (\frac{x}{\beta})^{\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-(\frac{\alpha+1}{\alpha})} + 2(\gamma_2 - \gamma_1) (\frac{x}{\beta})^{2\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-\frac{\alpha-2}{\alpha}} + 3(1-\gamma_2) (\frac{x}{\beta})^{3\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-\frac{\alpha-3}{\alpha}}]} \right) \left[\frac{\partial f(x)}{\partial \beta} \right] \right\} \quad (21)$$

$$\frac{d\log L(x, \alpha, \theta, \beta, \gamma_1, \gamma_2)}{d\gamma_1} = \left\{ \sum_{i=1}^n \ln \left(\frac{\frac{\alpha\theta}{\beta} [\gamma_1 (\frac{x}{\beta})^{\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-(\frac{\alpha+1}{\alpha})} + 2(\gamma_2 - \gamma_1) (\frac{x}{\beta})^{2\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-\frac{\alpha-2}{\alpha}} + 3(1-\gamma_2) (\frac{x}{\beta})^{3\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-\frac{\alpha-3}{\alpha}}]} \right) \left[\frac{\partial f(x)}{\partial \gamma_1} \right] \right\} \quad (22)$$

$$\frac{d\log L(x, \alpha, \theta, \beta, \gamma_1, \gamma_2)}{d\gamma_2} = \left\{ \sum_{i=1}^n \ln \left(\frac{\frac{\alpha\theta}{\beta} [\gamma_1 (\frac{x}{\beta})^{\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-(\frac{\alpha+1}{\alpha})} + 2(\gamma_2 - \gamma_1) (\frac{x}{\beta})^{2\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-\frac{\alpha-2}{\alpha}} + 3(1-\gamma_2) (\frac{x}{\beta})^{3\theta-1} (\alpha + (\frac{x}{\beta})^{\theta\alpha})^{-\frac{\alpha-3}{\alpha}}]} \right) \left[\frac{\partial f(x)}{\partial \gamma_2} \right] \right\} \quad (23)$$

The Maximum Likelihood Estimates ($\hat{\alpha}, \hat{\theta}, \hat{\beta}, \hat{\gamma}_1, \hat{\gamma}_2$) of The cubic transmuted Kappa distribution Equations $\frac{d\log L}{d\alpha} = 0, \frac{d\log L}{d\theta} = 0, \frac{d\log L}{d\beta} = 0, \frac{d\log L}{d\gamma_1} = 0, \frac{d\log L}{d\gamma_2} = 0$ The Equation (19), (20), (21), (22) and Equation (23) cannot be solved as they both are in locked forms. So we multiply the parameters of cubic transmuted Kappa distribution.

4. Application of Cubic Transmuted Kappa Distribution

"We compare our proposed distribution with the new cubic transmuted Kappa distribution, Kappa distribution. (khan and king, 2013) [9]. Here. By utilizing the maximum likelihood method, we estimate distribution parameters. The goodness of fit measures including the minus log-likelihood function (-LL), Akaike information criterion (AIC), Bayesian information criterion (BIC) and consistent Akaike information criterion (CAIC) are calculated to make a comparison among the fitted distributions. Generally, the lesser values of the above-mentioned measures, the better and the most appropriate fit to the information data set."

Data Set

Implementing the applied aspect on the real data that represents the working times until the malfunction of the axial spraying device measured in months, which was obtained with the help of the team of the Modern Irrigation and Mechanization Technologies Project, which amounts to (n=100) devices, and the following TABLE 1 shows the real data under study".

Table 1: The real data operating times until the malfunction of the axial spray device measured in months

0.4	1.6	2.2	2.7	2.9	3.4	3.7	4	4.4	5.3
0.4	1.7	2.4	2.7	3	3.4	3.7	4	4.4	5.4
0.3	1.7	2.4	2.7	3	3.4	3.8	4	4.7	5.5
1.1	1.8	2.4	2.7	3	3.5	3.8	4.1	4.7	5.6
1.1	1.9	2.5	2.7	3.1	3.5	3.8	4.2	4.8	5.7
1.4	1.9	2.5	2.8	3.1	3.5	3.9	4.2	4.8	6.1
1.5	1.9	2.6	2.8	3.1	3.6	3.9	4.2	4.9	6.8
1.5	1.9	2.6	2.8	3.1	3.6	3.9	4.3	5.1	7
1.6	2	2.6	2.8	3.1	3.6	3.9	4.3	5.1	7.2
1.6	2.1	2.6	2.9	3.2	3.7	3.9	4.3	5.2	8.5

Table 2: Descriptive statistics

Coefficients	Value
Mean	3.453 6
Variance	2.40943 4
Skewness	0.75169 1
Kurtosis	4.0360 4
StandardDeviation	1.5522357
Median	3.2
Max	8.1
Min	0.3

The maximum likelihood estimates and the goodness of fit measures are computed by the Wolfram Mathematical program software which is presented in the table below.

Table 3: Parameters "Estimates and Goodness" – of – fits by Akaka Information Criterion (AIC), Akaka Information Criterion Amended (AICC)

Distributions	Estimate of Parameters	-2log	AIC	AICC	BIC
Cubic Transmuted Kappa Distribution	524	202.778677	210.7776849574	211.447869498	—
Kappa Distribution	465.321582	332.246868	336.24686495	336.442870494	—

Note: The BIC values were not clearly identifiable in the provided text, so I have left them blank rather than guessing.

It is also clear that the distribution is preferable by cubic transmuted Kappa distribution representing the real data and its Applications:

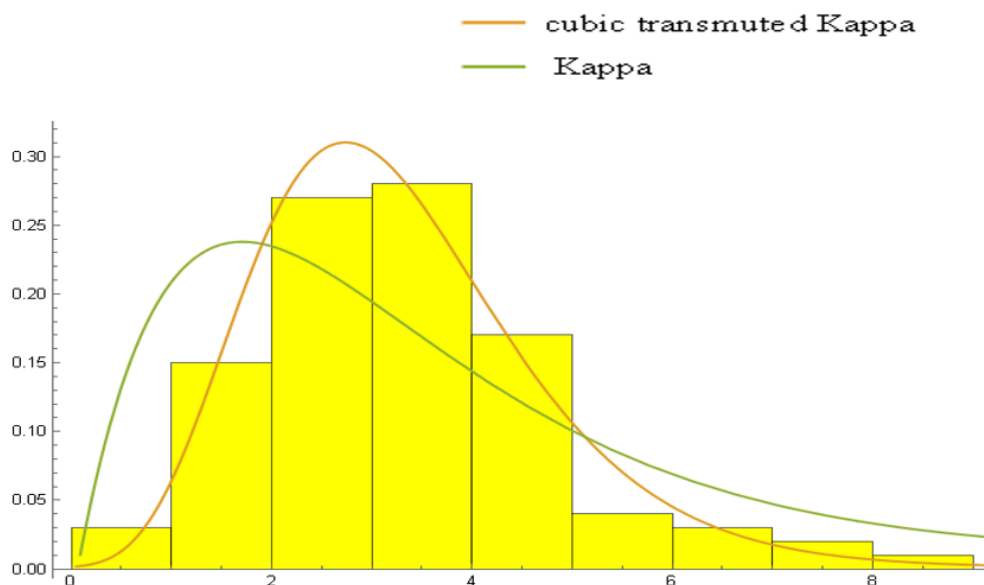


Fig 4: The Shape for the PDF cubic transmuted Kappa distribution in data and its Applications

5. Conclusion

We proposed and studied a new lifetime cubic transmuted Kappa distribution. The derived model is flexible with variable hazard rate. The Shapes of hazard rate function indicates that the new developed model is a competitive" model for other probability models having "constant, increasing and decreasing failure rate. Moreover, cubic transmuted Kappa distribution has special cases, which style it of divergent statistical significance from other models. We used real data for application purpose. The derived "model is more flexible and give better fit than "well-known existing models. So, we can conclude that our new distribution can be used an alternative models for muddling in various areas. For example, engineering, failure analysis, survival analysis, weather forecasting, extreme value theory, economics (income inequality) and others.

References

1. Afify AZ, Nofal ZM, Yousof HM. The transmuted Weibull Lomax distribution: properties and application. Pak J Stat Oper Res. 2015;11:135-152.
2. Ahmad A, Ahmad SP, Ahmad A. Transmuted inverse Rayleigh distribution: a generalization of the inverse Rayleigh distribution. Math Theory Model. 2014;4:90-98.
3. Aryal GR, Tsokos CP. On the transmuted extreme value distribution with application. Nonlinear Anal Theory Methods Appl. 2009;71:1401-1407.
4. Aryal GR. Transmuted log-logistic distribution. J Stat Appl Probab. 2013;2:11-20.
5. Ashour SK, Eltehiwy MK. Transmuted Lomax distribution. Am J Appl Math Stat. 2013;1:121-127.
6. Ashour SK, Eltehiwy MK. Transmuted exponentiated modified Weibull distribution. Int J Basic Appl Sci. 2013;2:258-269.
7. Cordeiro GM, Nadarajah S, Ortega EMM. The Kumaraswamy Gumbel distribution. Stat Methods Appl. 2012;21:139-168.
8. Elbatal I. Transmuted modified inverse Weibull distribution: a generalization of the modified inverse Weibull probability distribution. Int J Math Arch. 2013;4:117-122.
9. Khan MS, King R. Transmuted generalized inverse Weibull distribution. J Appl Stat Sci. 2013;20:15-31.
10. Kotz S, Nadarajah S. Extreme value distributions: theory and applications. London: Imperial College Press; 2000.
11. Lucena SEF, Silva AHA, Cordeiro GM. The transmuted generalized gamma distribution: properties and application. J Data Sci. 2015;13:187-206.
12. Mahmoud MR, Mandouh RM. Transmuted Frechet distribution. J Appl Sci Res. 2013;9:5553-5561.

13. Merovci F. Transmuted Rayleigh distribution. *Austrian J Stat.* 2013;42:21-32.
14. Nadarajah S. The exponentiated Gumbel distribution with climate application. *Environmetrics.* 2006;17:13-23.
15. Prudnikov AP, Brychkov YuA, Marichev OI. Integrals and series. Vol. 1. Amsterdam: Gordon and Breach Science Publishers; 1986.
16. Shaw WT, Buckley IRC. The alchemy of probability distributions: beyond Gram-Charlier and Cornish-Fisher expansions, and skew-normal or kurtotic-normal distributions [research report]. 2007.
17. Zhao YG, Lu ZH. Cubic normal distribution and its significance in structural reliability. *Struct Eng Mech.* 2008;28(3):263-280.

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